Is the Relation Between Volatility and Expected Stock Returns Positive, Flat or Negative?

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Abstract

Theoretical models, such as the CAPM, predict a positive relation between risk and return, but the empirical evidence paints a mixed picture. Positive, flat and negative relations have been reported in various empirical studies. In this paper we reconcile these seemingly conflicting results by showing how methodological choices can lead to different, or even opposite conclusions. In our 1963-2009 U.S. sample we find that the empirical relation between historical volatility and expected returns is negative, with an average quintile return spread of -3.7%.The relation becomes 2% less negative when small caps are excluded, but 3% more negative when compounding effects are taken into account. We also argue that the positive relation between volatility and expected return reported by some studies can be attributed to various kinds of look-ahead bias. Our results provide an empirical basis for low-volatility and minimum-variance investment approaches.

1. Introduction

The first empirical tests of the Capital Asset Pricing Model (CAPM) documented that the risk-return relation is flatter than predicted by theory. Studies by Black, Jensen and Scholes (1972), Fama and MacBeth (1973) and Haugen and Heins (1975) report positive alpha for low-beta and low-volatility stocks over the pre-1971 period. Twenty years later, the seminal Fama and French (1992) paper finds that the relation between risk and return turns flat, or even negative, over the 1963-1990 period. These findings are confirmed by Black (1993), Haugen and Baker (1991, 1996) and Falkenstein (1994), who look at similar or longer sample periods. More recently, Ang, Hodrick, Xing and Zhang (2006), Clarke, de Silva and Thorley (2010) and Baker, Bradley and Wurgler (2011) provide further evidence for a flat or negative relation between risk and return within the U.S stock market. Blitz and Van Vliet (2007) and Ang, Hodrick, Xing and Zhang (2009) show that these results also hold for international equity markets.

In contrast to this growing body of literature which documents a flat or negative relation between risk and return, some recent papers report opposite findings or dispute the robustness of earlier studies. Bali and Cakici (2008) argue that the significant negative relation reported by Ang, Hodrick, Xing and Zhang (2006) is driven by small cap stocks, and that if these stocks are excluded from the analysis the results become statistically insignificant. Martellini (2008) provides evidence that the empirical relation between total volatility and expected stock returns is positive, based on a sample consisting only of surviving stocks. Fu (2009) argues that one should focus on expected rather than historical volatility, and reports a positive relation between risk and return, using EGARCH models to estimate idiosyncratic volatility.

In general, the different outcomes and conclusions of the various studies may be related to different methodological choices. Exhibit 1 gives an overview of the main empirical results of key papers on the relation between risk and return. For each study we also report the choice for the universe, risk measure, return frequency, portfolio definition and return measure.

[Insert Exhibit 1 about here]

At first glance the reported annualized return spread between high- and low-risk stock portfolios differs by a wide margin, varying between -12.7% in Ang, Hodrick, Xing and Zhang (2006) and +57.5% in Fu (2009). Most empirical studies report a negative relation, with a median return spread across the different studies in Exhibit 1 of around -3% and an average return spread of around -1%.

In this paper we aim to reconcile the seemingly contradictory findings regarding the empirical relation between risk and expected stock returns. Specifically, we use three sample selection criteria, two risk measures, two return frequencies, two return measurements and two asset pricing models. Besides showing raw return differences, we also consider CAPM and 3-factor alpha spreads, because a flat relation between risk and raw return is likely to imply a negative alpha spread. For the sake of brevity we do not report the results for all possible combinations, but confine ourselves to the most important choices and empirical dimensions. We first examine results for sorting on 1-month and 5-year past total volatility and idiosyncratic volatility, for a broad universe of U.S. stocks and for a universe containing only the 1,000 largest stocks. We then examine the effects of using compounded (geometric) instead of simple (arithmetic) average returns. Finally, we examine the results of Martellini (2008) and Fu (2009), who both report a strong positive relation between risk and return.

Our first finding is that, in general, the average return spread across different settings amounts to -3.7% for all stocks and -1.5% for the largest 1,000 stocks. This implies that about 2% of the negative spread can be explained by the inclusion of small caps. We do not find much difference between idiosyncratic volatility versus total volatility, both measures yielding very similar results. For none of the settings the risk-return relation turns positive, as all the spreads we find are in a range between -5.5% and -0.5%.

Our second finding is that the risk-return relation inverts by an additional 3% by taking into account compounding effects when averaging returns. This result is consistent across all different empirical settings and in line with the expected impact based on textbook approximation formulas. Due to compounding effects, higher volatility leads to lower geometric average returns, especially lowering the returns of the most volatile stocks. This critical impact of the investment horizon on the slope of the risk-return relation is not yet emphasized in the literature.

Our third finding is that by using a sample with survivorship bias we can obtain results similar to those reported by Martellini (2008). The return spread jumps by about 7% if non-surviving stocks are excluded from the analysis, turning the risk-return relation positive. We find that survivorship bias particularly inflates the return of high-volatility stocks. Intuitively, this can be explained by realizing that high-risk stocks can generate very high returns if they are successful, but also very low returns if they fail. Many of the latter stocks are excluded from the analysis when considering only survivors.

Finally, we discuss the study of Fu (2009), who also reports a strong positive empirical relation between risk and return. This study differs from the previous ones by considering expected volatility (measured using an EGARCH model) instead of historical volatility. At first glance this different approach seems to have a critical impact on the relation between risk and return, but several studies have since shown that the results no longer hold after correcting for a look-ahead bias in Fu's estimation procedure.

In sum, the seemingly contradictory findings reported in the literature on the relation between volatility and expected returns can be explained by methodological choices with regard to sample selection (with or without small caps and look-ahead biases) and performance evaluation (compounded versus simple average returns). Other methodological choices do not appear to have a critical impact on the relation. We conclude that although theory predicts a positive relation between risk and return, the empirical relation is flat or negative. This result provides an empirical basis for low-volatility or minimum-variance investment approaches.

2. The Relation between Volatility and Expected Stock Returns

In order to analyze the relation between total volatility and expected stock returns we obtain daily and monthly return data from the Center for Research in Security Prices (CRSP) database. To avoid penny stocks driving the results, we exclude all stocks with a share price below $1. We then rank the stocks in our sample, every month from July 1963 until December 2009, on their 1-month total/idiosyncratic volatility and 60-month total/idiosyncratic volatility. We follow Ang, Hodrick, Xing and Zhang (2006) in the definition of idiosyncratic volatility and use data from the website of Kenneth French to control for systematic risk. We calculate equally weighted quintile portfolio returns over the subsequent month.[1]

[Insert Exhibit 2 about here]

Panel A in Exhibit 2 summarizes our results when we include all stocks. We find that the simple average annual return spread between the highest and lowest volatility quintile portfolios of stocks is -5.5% when sorting on 1-month total volatility and -5.2% when sorting on 1-month idiosyncratic volatility. The relation for both total and idiosyncratic risk measures is strongly negative, with statistically significant alphas between -8.3% (-3.29 t-stat) and -10.6% (-5.37 t-stat). When we consider longer-term (60-month) risk measures we also find negative return spreads, amounting to -1.9% for idiosyncratic and -2.1% for total volatility. Again we find very similar results for total volatility and idiosyncratic

[1] Unreported results for value-weighted portfolios lead to similar conclusions.

volatility. However, the longer-term risk measures seem to give less strong negative return spreads. Nevertheless, the alpha spread remains significant, varying between -5.9% (-1.99 t-stat) and -6.4% (-3.26 t-stat). As international evidence points in a different direction[2], and turnover is likely to be higher for strategies based on short-term risk models, we refrain from drawing firm conclusions on whether short-term or long-term risk measures produce stronger results. In sum, Panel A shows that on average the return spread is -3.7%, with negative and significant alpha spreads varying between -5.9% and -10.6%.

Anomalies are often stronger within the small cap segment. We therefore continue by examining the effects of excluding the smallest and least liquid stocks from our analysis. This should give more practically feasible results and addresses the Bali and Cakici (2008) critique. We follow the approach of Baker, Bradley and Wurgler (2011) by restricting the sample to the 1,000 largest stocks at each point in time. Panel B in Exhibit 2 shows that the simple return spread remains consistently negative, on average around -1.5%. Again we do not find much difference between idiosyncratic volatility and total volatility, and again shorter-term risk measures seem to give better results, although less pronounced than before. Interestingly, the CAPM alphas, which vary between -4.6% and -6.7%, are still statistically significantly negative, but some 3-factor alphas become insignificant. For example, the t-stat for 5-year idiosyncratic volatility drops to -1.0. This is in line with Bali and Cakici (2008), who report 3-factor alpha t-stats

[2] Exhibit 1 indicates that for international data, longer-term risk measures give stronger and more negative results than shorter-term risk measures. Specifically, Ang, Hodrick, Xing and Zhang (2009) use a short-term 1-month measure and report a return spread of about -4% for Europe/Asia compared to -12% for the U.S., while Blitz and Vliet (2007) report a return spread of -7% for Europe/Japan compared to -3% for the U.S. using a longer-term 3-year measure

between -0.5 and -1.7 (table 1, column 20% Market Share of their paper). Importantly, we find that the statistical significance disappears only in this very specific instance, i.e. in case the sample consists of large-caps only, the risk measure is long-term volatility, the evaluation measure is 3-factor alpha and returns are on a simple (arithmetic) basis.[3] For all other combinations of methodological choices we find statistically significant negative spreads.[4]

3. Simple versus Compounded Average Returns

Most empirical asset pricing studies implicitly assume a 1-month horizon. This choice for a monthly horizon is often not made explicit and simply a consequence of practical considerations, such as data format and data availability. In practice, however, investors have heterogeneous investment horizons, ranging from shorter than one day to multiple years. For systematic investment strategies, such as investing in low-volatility stocks, the horizon is typically well beyond one month, e.g. 3-5 years or even longer. It might therefore be more appropriate to look at, for example, 5-year returns. However, this would leave us only a small number of independent observations and empirical tests would lack statistical power. Another way to address the investment horizon issue is by considering geometric (compounded) instead of arithmetic (simple) average returns.

[3] A positive loading on HML, which carries a large positive premium, is the main reason for the loss of statistical significance. One can wonder, however, if this adjustment is entirely appropriate in light of the fact that we consider a sample consisting of the 1,000 largest stocks, whereas small-caps have a large weight in the definition of HML.

[4] Huang, Liu, Rhee and Zhang (2010) suggest that the anomalous relation between volatility and expected stock returns can be explained the well-known short-term reversal effect in stock returns. However, when we include a reversal factor (obtained from the Kenneth French website), alphas do not materially change, which is actually in line with the results reported by Huang *et al* in footnote 21 of their paper.

Arithmetic averaging ignores compounding effects, which are particularly important in case portfolios with very different volatility characteristics are being compared, as is clearly the case here. By considering geometric average returns we take into account that an investor in low-volatility stocks, who experiences a return of +20% followed by a return of -20%, will have a higher terminal wealth (-4%) than an investor in high-volatility stocks, who experiences a return of +40% followed by a return of -40% (-16%). With geometric average returns one implicitly assumes an evaluation horizon equal to the sample period, which, in our analysis, would imply an investment horizon of over 40 years (1963-2009). Because the truth is probably somewhere in the middle, we argue that it is important to consider both simple (short-term) and compounded (long-term) average returns, in particular for volatility-sorted portfolios.

Exhibit 3 shows how the main results in Exhibit 2 change as a result of considering compounded instead of simple average returns. We observe that the average spread between the return on the high-volatility and low-volatility quintile portfolios drops from -3.7% to -7.1%.[5] As expected, the relation between risk and return becomes even more strongly inverted. This is mainly driven by a lower return for the high-volatility (Q5) portfolio, which falls from 9.9% to 5.6%, i.e. about 4.3%. For the sample consisting of the largest 1,000 stocks we observe a similar drop of 3.7%. By contrast, the return of the low-volatility (Q1) portfolio decreases slightly, from 13.5% on an arithmetic basis to 12.7% on a geometric

[5] A textbook formula for approximating the effect of compounding is: geometric average = arithmetic average - variance / 2. As the volatilities of the quintiles range from around 12% to 30% (unreported), one would therefore expect the spread to drop by approximately 3.8% as a result of compounding. Our results are roughly in line with this number.

basis. On balance, the use of compounded returns lowers the spread by 3.4% for all stocks and 2.8% for the largest 1,000 stocks. These results are consistent with Baker, Bradley and Wurgler (2011), who also find that the return spread between high-volatility and low-volatility stocks is lowered by around 3% when considering compounded instead of simple average returns. When compounding effects are taken into account we observe that all spreads become more strongly negative and statistically significant.[6]

[Insert Exhibit 3 about here]

4. What about Martellini (2008) and Fu (2009)?

Our results so far indicate that the empirical relation between risk and return is negative, or flat at best. In contrast to this, Martellini (2008) reports a strong positive relation between past volatility and future stock returns. Whereas we find return spreads varying between -0.5% (simple returns, largest stocks only) and -8.9% (compounded returns, all stocks), Martellini (2008) reports a spread of +8.5%. The set-up of our empirical analysis so far is similar to his, except for the fact that he includes only surviving stocks in his sample.[7]

In order to examine if this explains the difference, we attempt to replicate his findings by taking the same sample period (1985-2004) and same volatility

[6] The asset pricing models employed in this study are single period models. Although theory does not prescribe a specific investment horizon, almost all empirical asset pricing studies use monthly data to estimate alphas and betas, which is consistent with a monthly investment horizon and consistent with using simple average returns. When compounded returns are employed to estimate returns and simple returns to calculate risk this is inconsistent with the asset pricing model although it gives an impression of how results would look like for longer investment horizons.

[7] Martellini (2008) explicitly mentions the use of a sample which consists of surviving stocks only.

estimate (10-year volatility). We then consider the results for two samples of stocks, one which includes all stocks in the CRSP universe (excluding penny stocks), and one which includes only those stocks which survived over the entire 1975-2004 period, as in the study of Martellini. Exhibit 4 shows that when the sample includes non-surviving stocks we find a negative return spread of -1.3% using simple returns and -3.9% using compounded returns, and alphas ranging from -3.6% (simple, FF) to -9.1% (compounded, CAPM). Although the statistical significance decreases due to the shorter sample period (20 years versus 45 years), this is in line with the previous results, despite using a longer volatility measure (120 months versus 1 or 60 months).

When we restrict our sample to survivors we observe a large drop in the number of stocks included in the analysis.[8] Panel B shows that, in line with the findings of Martellini (2008), the relation between past volatility and future stock returns indeed turns strongly positive for this specific sample. Specifically, we find a return spread between the top quintile of high-volatility stocks and the bottom quintile of low-volatility stocks amounting to +5.5%. In other words, by introducing a survivorship bias the return spread jumps by about 7%. Comparing this to the previous results, we conclude that the evidence for a positive relation between risk and return is entirely attributable to the choice of restricting the sample to surviving stocks only.

[Insert Exhibit 4 about here]

[8] By considering surviving stocks only, the number of stocks in the 1974-2005 sample drops from 2,160 to 694, which is very close to the number reported by Martellini (2008).

Another study which reports a strong empirical positive relation between risk and return is Fu (2009).[9] He argues that because volatilities are time-varying, it is more appropriate to look at expected volatilities instead of the more 'simple' risk measures we considered until now. Using EGARCH models to estimate expected idiosyncratic volatilities, he documents a positive relation between risk and return, with annualized decile spreads of 21% (value-weighted) and 57% (equal-weighted).

More recently, however, Guo, Kassa and Ferguson (2010) and Fink, Fink and He (2010) show that this strong positive relation between risk and return can be entirely attributed to a look-ahead bias in the parameter estimation procedure. Fink, Fink and He (2010) show that the relation between expected idiosyncratic volatility and returns turns negative after controlling for this bias (also see Exhibit 1). Guo, Kassa and Ferguson (2010) explain that, due to the nature of maximum likelihood estimation, the model parameters are estimated in such a way that the likelihood of extremely large observations is increased. As the evaluation month is included in the sample, evaluation months with large absolute returns will therefore have large forecasted volatilities. Because stock returns tend to be positively skewed, large positive returns will dominate large negative returns, leading to an upward bias in the relation between risk and return. Guo, Kassa and Ferguson (2010) replicate the results of Fu (2009), and show that the large positive premium disappears when avoiding the look-ahead bias.

[9] Other papers that look at EGARCH volatility forecasts include Huang, Liu, Rhee and Zhang (2010), Brockman,Schutte and Yu (2009) and Eiling (2008).

5. Explanations and implications

So, is the relation between historical volatility and expected return positive, flat or negative? Based on the results presented in this paper we conclude that whereas we would expect a positive relation, the relation has been negative (or at best flat) in practice, in particular for longer investment horizons when compounding effects come into play.

A closer look at the empirical return pattern shows a concave relation between risk and average return. Average returns do not consistently drop across the quintiles: first they go up slightly (by about 1%) and then after quintile 3 or quintile 4 average returns start to drop. Especially the 20% most risky stocks have the most anomalous returns. Although this non-linear pattern becomes less pronounced for the largest stocks or when using compounded returns it remains present. On the other hand, the alphas across the quintiles (unreported) do show a monotonically decreasing pattern, although still with the biggest and most significant drop for the alpha of the high-volatility stocks.

In the literature several explanations have been put forward to explain these anomalous results. We briefly mention them here. First, Black (1993) argues that investors face leverage restrictions which tend to flatten the risk-return relation. De Giorgi and Post (2011) extend this reasoning by showing that a restriction on short-selling distorts the risk-return relation in a non-linear, concave way. A second explanation is that investors with a relative return perspective flatten the risk-return relation; see Blitz and Vliet (2007), Falkenstein

(2010) and Baker, Bradley and Wurgler (2011). Although low-volatility stocks may be attractive in terms of alpha and Sharpe ratio, they may still be unattractive for investors with relative return objectives. Third, the gaming effect of Siri and Tufano (1998) implies that mutual fund managers have an incentive to buy high-risk stocks and neglect low-risk stocks, also causing a flattening of the risk-return relation. Fourth, if investors perceive stocks as lottery tickets this may cause high-risk stocks to become overpriced, which can even make the risk-return relation turn negative; see, e.g., Barberis and Huang (2008). Interestingly, only the latter explanation can explain a negative relation, while the other explanations can only explain a flat relation.

What are the practical implications of a flat or negative relation between risk and return for investors? Black (1993) already argued that investors should tilt their portfolios towards low-beta stocks in order to achieve superior risk-adjusted returns. The leverage restriction can be lifted relatively easily within the asset mix, by reducing the allocation to bonds and increasing the allocation to low-volatility equities. The documented failure of the CAPM has also fueled the need for alternative, 'smarter' indices. Over the years, several alternative indices have been proposed, aimed at providing a better risk/return profile than the capitalization-weighted index.[10] Our results argue against the use of alternative indices which explicitly assume a positive relation between risk and return, as in

[10] Examples of alternative indices are Fernholz, Garvy, and Hannon (1998), who propose to use a measure of the distribution of capital in an equity market, which they call diversity, as a weighting factor for the construction of an index and Arnott, Hsu and Moore (2005), who introduce the concept of fundamental indices, in which the weight of each stock is set in proportion to its fundamentals, such as book value, sales, cash-flow and dividends.

Martellini (2008).[11] Minimum-variance indices, on the other hand, implicitly capitalize on the flat or negative relation between risk and return which is found empirically. Minimum-variance indices are inspired by the early work of Haugen and Baker (1991), with more recent evidence being provided by, for example, Clarke, de Silva and Thorley (2006). Algorithms that optimize a portfolio for minimum variance may not solely concentrate on selecting stocks with a low volatility, as stocks with low cross-correlations may also be attractive in the optimization process. However, they do have a strong preference for low-volatility stocks, as shown by, for example, Scherer (2010). As such, it is not surprising that the reported empirical performance characteristics of minimum-variance indices are broadly similar to those of simple quintile portfolios consisting of low-volatility stocks; see, for example, Haugen and Baker (1991) and Clarke, de Silva and Thorley (2006). Contrary to many other active investment approaches, low-volatility investment approaches have in common that they aim to obtain a higher Sharpe ratio primarily by focusing on reducing volatility, rather than on increasing return.

6. Conclusions

[11] Martellini (2008) argues that alternative indices should have a solid foundation in modern portfolio theory, which states that a mean-variance utility-maximizing agent should hold the portfolio with the highest reward-to-risk ratio, also known as the tangency portfolio or maximum Sharpe ratio portfolio. He then derives this tangency portfolio by using sophisticated estimators for the variance-covariance matrix of stock returns and for expected stock returns, and empirically shows that this tangency portfolio outperforms its capitalization-weighted counterpart on a risk-adjusted basis. An important assumption in this regard is that the relation between the volatility of a stock and its expected return is positive, but we have shown that the empirical evidence in support of this assumption can be attributed entirely to the use of a sample which contains only surviving stocks.

In theory the relation between volatility and expected stock returns should be positive, but the empirical evidence suggests that the relation is flat or even negative in reality. We have reconciled the conflicting empirical results by showing how methodological choices can lead to different, or even opposite conclusions. In our 1963-2009 U.S. sample we find that the empirical relation between historical volatility and expected returns is negative, with an average quintile return spread of -3.7%.The relation becomes 2% less negative when small caps are excluded, but 3% more negative when compounding effects are taken into account. We also show that studies which have reported a strong positive relation between volatility and expected return consider strategies which are not feasible in practice due to look-ahead biases. Our results provide an empirical basis for low-volatility and minimum-variance investment approaches.

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Exhibit 1: Literature Overview

This table provides an overview of the main findings of the studies which examine the empirical relation between risk and return.

Study	Universe	Sample period	Variable	Data freq.*	Portfolios **	Weighting	Return basis	High minus low risk raw return spread
Fama & French (1992)	CRSP all, ex financials	1963:07-1990:12	5Y beta	M	decile	equal	simple	-2.4%
Black (1993)	NYSE listed	1931:01-1991:12	5Y beta	M	decile	equal	simple	-4.0%
Falkenstein (1994)	CRSP, price > $5	1965:01-1992:12	2Y variance	M	quartile	equal	simple/compounded	-2.6% / -3.9%
Ang et al. (2006)	CRSP all / NYSE listed	1963:07-2000:12	1M ivolatility	D	quintile	value	simple	-12.7% / -7.9%
Blitz & van Vliet (2007)	FTSE World / US	1985:12-2006:01	3Y volatility	W	decile	equal	compounded	-5.9% / -3.1%
	FTSE Europe / Japan	1985:12-2006:01	3Y volatility	W	decile	equal	compounded	-6.0% / -7.5%
Bali & Cakici (2008)	CRSP all	1963:07:2004:12	1M ivolatility	D	quintile / 20%share	value	simple	-11.2% / -0.5%
	NYSE listed	1963:07:2004:12	1M ivolatility	D	quintile / 20%share	value	simple	-2.8% / +1.1%
Martellini (2008)	CRSP survivors 1975-2004	1985:01:2004:12	10Y volatility	M	quintile	equal	simple	+8.5%
Ang et al. (2009)	MSCI Europe/Asia	1980:09-2003:12	1M ivolatility	D	quintile	value	simple	-4.9% / -3.2%
Fu (2009)	CRSP all	1963:07-2006:12	E(ivolatility)	D	decile	value	simple	+21.0%
	CRSP all	1963:07-2006:12	E(ivolatility)	D	decile	equal	simple	+57.5%
Fink, Fink, He (2010)	CRSP small / large	1963:07-2008:12	E(ivolatility)	D	quintile	value	simple	-3.8% / -0.2%
Clarke et al. (2010)	CRSP all	1931:01:2008:12	5Y ivov/vol	M	tertile (double sort)	equal	simple	+0.5% / +0.8%
Baker et al. (2011)	CRSP all	1968:01-2008:12	5Y volatility	M	quintile	equal	simple/compounded	-6.9% / -11.2%
	CRSP top 1000	1968:01-2008:12	5Y volatility	M	quintile	equal	simple/compounded	+0.9% / -2.1%

* D = Daily, W = weekly, M = monthly

** 20% share means that each quintile consist of 20% of the total market capitalization (instead of 20% of the number of stocks).
Double sort is similar to Fama-French procedure with first a split based on the median NSYE market capitalization and subsequently three portfolios sorted on risk.

Exhibit 2: Arithmetic Returns of Stock Portfolios Sorted on Volatility

This table shows results of quintile portfolios based on sorting stocks on their past volatility (standard deviation) using monthly and daily return data. The 20% least-volatile stocks are assigned to Q1 and the 20% most-volatile stocks are assigned to Q5, beginning in July 1963 and ending in December 2009. We employ a total volatility (TV) risk measure, Idiosyncratic Volatility (IV) risk measure estimated on 30-days of data (1M) and 60-months of data (5Y). Panel A includes all stocks in the CRSP database at each portfolio formation month. Panel B includes only the largest 1,000 stocks as measured at formation date. The return difference and alphas are measured for the low-minus-high Q1-Q5 portfolio and the t-stats are in brackets.

Panel A: ALL

	Q1	Q2	Q3	Q4	Q5	Q5-Q1	CAPM alpha	(t-stat)	FF alpha	(t-stat)
TV_1m	13.4%	16.2%	17.0%	16.0%	7.9%	-5.5%	-9.3%	(-3.43)	-10.6%	(-5.37)
IV_1m	13.8%	15.6%	16.8%	15.6%	8.7%	-5.2%	-8.3%	(-3.29)	-9.7%	(-5.20)
TV_5y	13.5%	14.7%	15.3%	15.3%	11.6%	-1.9%	-6.3%	(-2.10)	-6.4%	(-3.26)
IV_5y	13.4%	15.0%	15.4%	15.2%	11.3%	-2.1%	-5.9%	(-1.99)	-6.0%	(-3.05)
Average	13.5%	15.4%	16.1%	15.6%	9.9%	-3.7%	-7.4%		-8.2%	

Panel B: Top 1,000

	Q1	Q2	Q3	Q4	Q5	Q5-Q1	CAPM alpha	t-stat	FF alpha	t-stat
TV_1m	11.7%	13.4%	13.7%	14.2%	9.7%	-2.0%	-6.4%	(2.79)	-4.2%	(2.38)
IV_1m	12.4%	13.8%	13.7%	14.1%	9.5%	-3.0%	-6.7%	(3.08)	-4.7%	(3.06)
TV_5y	11.6%	13.3%	13.4%	13.2%	11.0%	-0.5%	-5.3%	(2.07)	-2.2%	(1.34)
IV_5y	12.0%	12.9%	13.3%	12.8%	11.6%	-0.4%	-4.6%	(1.86)	-1.6%	(1.00)
Average	11.9%	13.4%	13.6%	13.6%	10.4%	-1.5%	-5.7%		-3.2%	

Exhibit 3: Geometric Returns of Stock Portfolios Sorted on Volatility

This table shows results of quintile portfolios based on sorting stocks on their past volatility (standard deviation) using monthly and daily return data. The 20% least-volatile stocks are assigned to Q1 and the 20% most-volatile stocks are assigned to Q5, beginning in July 1963 and ending in December 2009. We employ a total volatility (TV) risk measure, Idiosyncratic Volatility (IV) risk measure estimated on 30-days of data (1M) and 60-months of data (5Y). Panel A includes all stocks in the CRSP database at each portfolio formation month. Panel B includes only the largest 1,000 stocks as measured at formation date. The return difference and alphas are measured for the low-minus-high Q1-Q5 portfolio and the t-stats are in brackets.

Panel A: ALL

	Q1	Q2	Q3	Q4	Q5	Q5-Q1	CAPM alpha	(t-stat)	FF alpha	(t-stat)
TV_1m	12.6%	14.7%	14.9%	13.0%	3.7%	-8.9%	-11.8%	(-4.57)	-12.6%	(-6.73)
IV_1m	12.8%	14.1%	14.7%	12.8%	4.7%	-8.2%	-10.6%	(-4.40)	-11.5%	(-6.49)
TV_5y	12.7%	13.3%	13.2%	12.2%	6.9%	-5.8%	-9.2%	(-3.19)	-9.1%	(-4.74)
IV_5y	12.6%	13.5%	13.3%	12.2%	6.9%	-5.6%	-8.6%	(-3.00)	-8.5%	(-4.42)
Average	12.7%	13.9%	14.0%	12.5%	5.6%	-7.1%	-10.0%		-10.4%	

Panel B: Top 1,000

	Q1	Q2	Q3	Q4	Q5	Q5-Q1	CAPM alpha	t-stat	FF alpha	t-stat
TV_1m	10.9%	12.2%	12.2%	12.1%	6.1%	-4.8%	-8.2%	(-3.60)	-6.2%	(-3.55)
IV_1m	11.4%	12.5%	12.2%	11.9%	5.8%	-5.6%	-8.5%	(-3.96)	-6.7%	(-4.36)
TV_5y	10.8%	12.1%	11.8%	11.1%	7.2%	-3.6%	-7.3%	(-2.90)	-4.6%	(-2.78)
IV_5y	11.1%	11.7%	11.7%	10.7%	8.0%	-3.1%	-6.4%	(-2.62)	-3.7%	(-2.38)
Average	11.1%	12.1%	12.0%	11.5%	6.8%	-4.3%	-7.6%		-5.3%	

Exhibit 4: Impact of Survivorship Bias

This table shows results of quintile portfolios based on sorting stocks on their past 10 year volatility (standard deviation) using monthly return data. The 20% least-volatile stocks are assigned to Q1 and the 20% most-volatile stocks are assigned to Q5, beginning in January 1985 and ending in December 2004. Panel A includes all stocks in the CRSP database at each portfolio formation month. Panel B includes only the surviving 694 stocks over the 1975-2004 period. The return difference and alphas are measured for the low-minus-high Q1-Q5 portfolio and the t-stats are in brackets.

Panel A: ALL stocks

	Q1	Q2	Q3	Q4	Q5	Q5-Q1	CAPM alpha	(t-stat)	FF alpha	(t-stat)
Arithmetic	15.9%	16.0%	17.2%	17.1%	14.6%	-1.3%	-7.1%	(-1.6)	-3.6%	(-1.3)
Geometric	15.3%	14.9%	16.0%	15.3%	11.4%	-3.9%	-9.1%	(-2.1)	-5.6%	(-2.1)

Panel B: Surviving stocks only

	Q1	Q2	Q3	Q4	Q5	Q5-Q1	CAPM alpha	(t-stat)	FF alpha	(t-stat)
Arithmetic	14.5%	15.7%	15.7%	16.7%	20.0%	5.5%	0.0%	(0.0)	0.6%	(0.3)
Geometric	13.9%	14.8%	14.5%	15.2%	17.7%	3.8%	-1.2%	(-0.4)	-0.3%	(-0.1)